

# Fuzzy Markov Model for Web server Queues

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## Abstract:

In this paper queuing network model which can be used for web traffic networks has been studied. The model is an open network of infinite server queues where client arrive according to a non- homogenous fuzzy Poisson process. The interaction of the client with server is described by a Fuzzy Markov Renewal Process. The changing attribute of the customer in the customer are driven by continuous time Markov chain and therefore change as they move through the network. The transient and limited number of customers in disjoint sets of servers and attributes are investigated. These turns out to be independent fuzzy Poisson random variable. The covariance of number of clients in 2 sets of servers and attributes at different time epochs has also been discussed.

**Index Terms**— Continuous time Markov chain (CTMC), independent fuzzy Poisson random variable, Non Homogeneous Fuzzy Poisson Process(NHFPP), Fuzzy Markov Model, Fuzzy Markov renewal Process, Fuzzy Semi Markov kernel, Covariance.

## 1 Introduction:

In this paper traffic model for a web multimedia wireless network has been considered. Characterized by interaction between client server network and fuzzy web traffic aspects. We suppose that there exists a finite set of servers. This choice avoids the detailed location description of client interaction with server in order to obtain an integrated manageable model. Since in real time the arrival rate of call vary significantly over time we consider that clients arrive according to a non homogeneous fuzzy Poisson process. The successive server visited by a client form a discrete time Markov chain and the server residence time have a general distribution which depends on the call being visited as well as both the previous server visited and the next server to be entered.

A new method have been introduced for finding fuzzy system reliability using fuzzy profust reliability theory[2]. In [5] the theoretical aspects of modeling the uncertainty associated with links are discussed, whereas in [1] unified fuzzy Markov model has been used in a communication network .

Web is being used by all short of people for different purpose.

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However with the aim of making our system more clear we present ours as a queuing network. Here we speak about

open network model of infinite server queues where client transition state in the network in according a Fuzzy Markov Renewal process and arrival pattern of the client NHFPP. We also take into account the characteristic of the client according to a CTMC.

## 2 Queueing Model:

The model consists of a fuzzy web system of infinite work stations. A client entering for 'job n' to k<sup>th</sup> station. Let  $D = I \cup M \cup O$  denotes the set of nodes of the network where, I - denotes the input node which will be visited by the client only when they enters the network  
O - denotes the output node which will be visited by the client when he leaves the network, once he leaves the system he is supposed to reenter the network only through I  
M - Denoted the intermediate nodes where the client may stay and has a freedom of moving different area's in search of information he requires.

## 3 Preliminaries:

A fuzzy set is the generalization of the crisp sets. A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to a degree to which that individual is similar or compatible with the concept represented by the fuzzy set.

Let  $\tilde{A}$  be a fuzzy set, the membership function [6]  $\mu_{\tilde{A}}(x) \in [0,1]$  is evaluated for  $\tilde{A}$  at  $x \in R$  where  $[0,1]$  denotes the interval of real number from 0 to 1 including 0 and 1. Then the fuzzy sets are the subsets of real number system.

Let  $\Gamma$  be the universe of discourse and  $\psi$  be the power set of  $\Gamma$ . Then the possibility measure  $\sigma$  is a mapping defined as follows [6]:

$$\sigma : \psi \rightarrow [0, 1]$$

such that the following properties holds

i.  $\sigma(\emptyset) = 0$  and  $\sigma(\Gamma) = 1$

ii.  $\sigma(\cup A_i) = \sup(\sigma(U A_i))$

for every arbitrary collection  $A_i$  of  $\psi$ . The triplet  $(\Gamma, \psi, \sigma)$  is called as possibility space.

**4 Fuzzy Markov model:**

A fuzzy Markov model [6] is the model which has a finite number of states  $S_i, i = 1,2,3,...m$  at each transition  $n = 1,2,3,...m$  together with fuzzy possibilities [2]

$$\tilde{p}_{ij} = \sigma[X_{n+1} = S_j | X_n = S_i].$$

Let  $E = \{1,2,3,...m\}$  be the state space and  $(\Gamma, \psi, \sigma)$  be a possibility space. Then

$X_n$  represents the state and the  $n$ th transition and

$T_n$  represents the time of  $n$ th transition.

The process  $(X_n, T_n), n \in N$  is called non homogenous fuzzy Markov Renewal process if

$$\sigma[X_n = j, T_n \leq t | X_0, X_1, \dots, X_{n-1} = i, T_0, T_1, \dots, T_{n-1} = s] \\ = \sigma[X_n = j, T_n \leq t | X_{n-1} = i, T_{n-1} = s] ; i \neq j.$$

**4.1 Mobility of the customer:**

Let  $X_i^n$  denotes location node of the customer at  $T_i^n$  time.

$$\{(X_i^n, T_i^n), n \geq 0\} \sim \{(X^n, T^n), n \geq 0\} \text{ -----(1)}$$

where

$X_n$  represents the steps at the  $n$  transition

$T_n$  represents the time at the  $n$  transition

The process  $(X_n, T_n) n \in N$  is called non- homogeneous Fuzzy Markov Renewal process if

$$\sigma(X_{n+1} = j, T_{n+1} \leq t | X_0, X_1, X_2, \dots, X_n = i, T_0, T_1, \dots, T_n) \\ = \sigma(X_{n+1} = j, T_{n+1} \leq t | X_n = i); \text{ for } i \neq j$$

The Transition distribution measure follows a homogenous fuzzy semi markov kernel [2].

$$\tilde{Q}_{ij} = \sigma(X_{n+1} = j, T_{n+1} \leq t | X_n = i)$$

$$\tilde{r}_{ij}(t) = \lim_{n \rightarrow \infty} \tilde{Q}_{ij}(s, t), i, j \in E, j \neq i$$

$\tilde{r}_{ij}(t)$  represent the possibility of the client making his next transition to state  $j$  given that he entered state  $i$  at time  $t$ . However before the entrance to  $j$  the process holds for a time  $s$  in state  $i$ .

For  $i, j \in E$ . Define  $[\tilde{G}_{ij}(t), t \geq 0]$  as the fuzzy distribution function of the time spent by the customer at the visiting node  $i$ , given that its next visiting node is  $j$  [2].

$$\therefore \tilde{G}_{ij}(t) = \begin{cases} \frac{\tilde{Q}_{ij}}{\tilde{r}_{ij}(t)}, & \text{if } \tilde{Q}_{ij} \neq 0 \\ 1, & \text{if } \tilde{Q}_{ij} = 0 \end{cases}$$

where

$$\tilde{G}_{ij}(t) = \frac{\tilde{Q}_{ij}}{\tilde{r}_{ij}(t)} = \sigma(T_{n+1} - T_n \leq t | X_{n+1} = j, X_n = i) \text{ if } \tilde{p}_{ij} > 0.$$

Let  $N^* = \inf \{n \geq 1; X_n, t > 0\}$  denotes the number of node transition made by an arbitrary customer before leaving the network.

$T_{N^*}$  denotes the time of departure so that  $X_{N^*}$  is the outside node reached by the customer at departure.

$Y^l(t)$  denotes the location node of the  $l^{th}$  customer at  $t$  unit of time after arriving the network.

$$\{Y^l(t), t > 0\} \sim \{Y(t), t > 0\}$$

Where  $\{Y(t), t > 0\}$ , being the minimal semi markov process associated to  $\{(X_n, T_n), n \geq 0\}$  defined by

$$Y^l(t) = \begin{cases} X_n, & T_n \leq t \leq T_{n+1} \\ X_{N^*}, & t \geq T_{N^*} \end{cases} \text{ for all } t > 0 \text{ and } i \in l, j \in 0.$$

We have

$$\tilde{p}_{ij}(t) = \sigma[Y(t) = j | Y(0) = i] = \min[\tilde{p}_{ij}(t), \tilde{r}_{ij}(t)]$$

**4.2 Customers Attributes:**

Suppose that each customer in the network has associated an attributes that many change with time. Let  $A^l(t)$  denotes the attributes of  $l^{th}$  customers at  $t$  unit of time after arriving the network[7].

Suppose if

$\{A^l(t), t > 0\} \sim \{A(t), t > 0\}$  with  $\{A^l(t), t > 0\}$  being a continuous time markov chain with finite state space  $S$ .

For  $t \geq 0$  and  $i, j \in S$  we have

$$\tilde{q}_i(t) = P[A^l(t) = i]$$

$$\tilde{q}_{ij}(t) = P[\{A^l(t) = j | A^l(0) = i\}]$$

Let us also suppose that the attribute of the customer and their locations in the network are independent (i.e)

$$\{A^l(t); t \geq 0, l \geq 1\} \perp \{Y^l(t), t > 0, l \geq 1\}$$

**Transient Analysis**

The arrival process of the customer to the network[7] Using time dependent coloring Poisson process we determine.

**Theorem 1:**

Suppose that a customer enters to the network at time  $s$  is placed in cell  $k$  with probability  $\tilde{p}_k(s), k \in K$  be finite and independent on other arrivals, For  $t \geq 0$  and  $k \in K$ , let  $M_k(t), t \geq 0$  denotes the number of customer placed in cell  $k$  unit time ' $t$ '. Then the point process  $\{\mu_k(t), t \geq 0\}, k \in K$  are independent Non- homogenous fuzzy Poisson Process with intensity measure  $\mu_k(dt) = \tilde{p}_k(t). \lambda(t) dt, t \geq 0, k \in K$ . For all  $t \geq 0$  the random varivable  $\{M_k(t), k \in K\}$  are independent and  $M_k(dt) = \text{Poisson}(\int_0^t \mu_k(ds)), k \in K$

**Note:**

Let us suppose  $\{s_l^1, l \geq 1\}$  denotes the sequence of arrival time to the network through input node  $l$  and let  $\{N_l(t), t \geq 0\}$  denotes the associated counting process.

Then by the theorem 1 with  $k = I$  and  $\tilde{P}_i(s) = \tilde{v}_i(s)$  we conclude that  $\{N_i(t), t \geq 0\} i \in I$  are independent non homogenous fuzzy Poisson process with intensity measure [4].

$$\Lambda_i(dt) = \tilde{v}_i(t)\lambda(t) dt, i \in I \text{ and } t \geq 0.$$

Let  $B \subseteq I \times E$ ,  $Y_\beta(t)$  denotes the number of customers whose pair of location node and attribute at time  $t$  belongs to the set  $B$ . Then the number of customer that are in node  $j$  with attribute  $\beta$  at the time  $Y_{(j,\beta)}(t)$ .

**Theorem 2:**

For  $t \geq 0$ , the random variables  $Y_{(j,\beta)}(t) j \in D, \beta \in A$  are independent and

$$Y_{(j,\beta)}(t) \sim \text{Poisson} \left[ \sum_{i \in I} \int_0^t \tilde{q}_\beta(t-s) \cdot \tilde{P}_{ij}(t-s) \cdot \Lambda_i(ds) \right]$$

**Proof:**

Consider  $t \geq 0$  If the  $l$  th customer enters the network through node  $I$  at the time  $s, 0 \leq s \leq t$ , then it will be located in node  $j$  and have attribute  $\beta$  at time  $t$  if

$Y^l(t-s) = j$  and  $A^l(t-s) = \beta$  is independent of other customers. Thus for a customer arriving at time  $s, 0 \leq s \leq t$ , and having input node  $\alpha$ , the probability of being located in node  $j$  with attribute  $\beta$  at time  $t$  is only function of its arrival time  $s$  and is equal to  $\tilde{v}_{ij}^\beta(t)$  given below

$$\begin{aligned} \tilde{v}_{ij}^\beta(s) &= \tilde{P}((Y(t-s) = j, A(t-s) = \beta) | Y(0) = i) \\ &= \tilde{p}_{ij}(t-s) \tilde{q}_\beta(t-s). \end{aligned}$$

For  $i \in I, j \in M$  and  $\beta \in A$ , we identify a mark  $(i, j, \beta)$  customers. Therefore using theorem 1 with  $k = I \times M \times A$  and

$$\tilde{P}_{(i,j,\beta)}(s) = \begin{cases} \tilde{v}_i(s) \tilde{v}_{ij}^\beta(s), & 0 \leq s \leq t \\ 0, & \text{otherwise} \end{cases}$$

for  $(i, j, \beta) \in K$ , we conclude that  $X_{(i,j,\beta)}(t)$  are independent fuzzy Poisson random variable with mean

$$\begin{aligned} E[X_{(i,j,\beta)}(t)] &= \int_0^t \tilde{P}_{(i,j,\beta)}(t) \Lambda(ds) \\ &= \int_0^t \tilde{q}_\beta(t-s) \cdot \tilde{P}_{ij}(t-s) \cdot \Lambda_i(ds) \end{aligned}$$

Thus the number of customers that have a given attribute and are located in a fixed node of the network at time  $t$  are independent fuzzy Poisson random variable.

Let  $\{B_m, m = 1, 2, 3, \dots, M\}$  be a partition of the set  $M \times A$ , the set of all pairs of a node and an attribute. The number of customers whose pairs of location node and attributed at time  $t$  belongs to the set  $B_m$  is

$$Y_{B_m}(t) = \sum_{(j,\beta) \in B_m} Y_{(j,\beta)}(t), 1 \leq m \leq M, t \geq 0.$$

**Corollary:**

If  $\{B_m, m = 1, 2, 3, \dots, M\}$  be a partition of the set  $M \times A$  then the fuzzy random variables then the random variable  $Y_{B_1}(t), Y_{B_2}(t), \dots, Y_{B_m}(t)$  are independent and

$$Y_{B_m}(t) \sim P \left[ \sum_{i \in I} \sum_{(j,\beta) \in B_m} \int_0^t \tilde{q}_\beta(t-s) \cdot \tilde{P}_{ij}(t-s) \cdot \Lambda_i(ds) \right]$$

for  $1 \leq m \leq M$  and  $t \geq 0$

Thus it has been shown that the number of customers whose pair of location node and attribute at time  $t$  belongs to each one of disjoint sets of pairs of a node and an attribute are independent fuzzy Poisson random variable.

**Covariance with respect to time:**

Here the covariance of the number of customers with given (single or multiple) pairs of location node and attribute at different times is determined.

**Theorem 3:**

For  $t, (t+h) \geq 0, j_1, j_2 \in M, \beta_1, \beta_2 \in A$

$$\text{Cov}(Y_{(j_1,\beta_1)}(t), Y_{(j_2,\beta_2)}(t+h))$$

$$= \sum_{i \in I} \int_0^t \tilde{q}_{\beta_1}(t-s) \cdot \tilde{q}_{\beta_2}(t+h-s) \cdot \tilde{P}[(Y(t-s) = j_1, Y(t+h-s) = j_2 | Y(0) = i) \Lambda_i(ds)]$$

**Proof:**

Let  $t, (t+h) \geq 0, j_1, j_2 \in M, \beta_1, \beta_2 \in A$

$X_{(j_1,\beta_1)(j_2,\beta_2)}(t, t+h)$  denotes number of customers that he is in the node  $j_1$  with attribute  $\beta_1$  at the time  $t$  and in the node  $j_2$  with attribute  $\beta_2$  at time  $t+h$ . We conclude that  $X_{(j_1,\beta_1)(j_2,\beta_2)}(t, t+h)$  is a fuzzy Poisson random variable with mean.

$$\begin{aligned} E[X_{(j_1,\beta_1)(j_2,\beta_2)}(t, t+h)] &= \\ &= \sum_{i \in I} \int_0^t \tilde{q}_{\beta_1}(t-s) \cdot \tilde{q}_{\beta_2}(t+h-s) \cdot \tilde{P}[(Y(t-s) = j_1, Y(t+h-s) = j_2 | Y(0) = i) \Lambda_i(ds)] \end{aligned}$$

**If  $N_1, N_2, N_3$  are independent fuzzy Poisson random variable then**

$$\text{Cov}[N_1 + N_2, N_1 + N_3] = E[N_1]$$

$$\therefore \text{Cov}[Y_{(j_1,\beta_1)}(t), Y_{(j_2,\beta_2)}(t+h)] = E[X_{(j_1,\beta_1)(j_2,\beta_2)}(t, t+h)]$$

$$\begin{aligned} Y_{(j_1,\beta_1)}(t) &= X_{(j_1,\beta_1)(j_2,\beta_2)}(t, t+h) + Y_{(j_1,\beta_1)}(t) \\ &\quad - X_{(j_1,\beta_1)(j_2,\beta_2)}(t, t+h) \end{aligned}$$

$$Y_{(j_2, \beta_2)}(t) = X_{(j_1, \beta_1)(j_2, \beta_2)}(t, t+h) + Y_{(j_2, \beta_2)}(t+h) - X_{(j_1, \beta_1)(j_2, \beta_2)}(t, t+h)$$

Hence the three random variables in the second member of last two equations are independent fuzzy Poisson random variable.

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#### Conclusion:

The transient number of customer that have a given attributed and are located in a fixed node are shown to be independent Poisson random variable.

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